Suppose an 8-bit word stored in memory is 00111101 with the data bit D1 in right most position.

Using the Hamming code algorithm, determine:

a) length of check bits to be stored with the data word
b) what check bits would be stored in memory with the data word
Exercise 1 - Solution

a) length of check bits to be stored with the data word

• Length of Hamming code \(K\) should satisfy:
  \[ 2^K - 1 \geq M + K, \text{ where } M = 8 \text{ (length of data)} \]
  
  ➤ If \(K = 2\) then \(2^2 - 1 \geq 8 + 2\)
  ➤ If \(K = 3\) then \(2^3 - 1 \geq 8 + 3\)
  ➤ If \(K = 4\) then \(2^4 - 1 \geq 8 + 4\)

• So, \(K = 4\).

Exercise 1 - Solution

b) what check bits would be stored in memory with the data word

\[
\begin{array}{cccccccc}
D8 & D7 & D6 & D5 & D4 & D3 & D2 & D1 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\
\end{array}
\]

<table>
<thead>
<tr>
<th>Bit positions</th>
<th>12</th>
<th>11</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position number</td>
<td>1100</td>
<td>1011</td>
<td>1010</td>
<td>1001</td>
<td>1000</td>
<td>0111</td>
<td>0110</td>
<td>0101</td>
<td>0100</td>
<td>0011</td>
<td>0010</td>
<td>0001</td>
</tr>
<tr>
<td>Data bit</td>
<td>D8</td>
<td>D7</td>
<td>D6</td>
<td>D5</td>
<td>D4</td>
<td>D3</td>
<td>D2</td>
<td>D1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Check bit</td>
<td>C4</td>
<td>C3</td>
<td>C2</td>
<td>C1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
C1 &= D1 \oplus D2 \oplus D4 \oplus D5 \oplus D7 = 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0 = 1 \\
C2 &= D1 \oplus D3 \oplus D4 \oplus D6 \oplus D7 = 1 \oplus 1 \oplus 1 \oplus 1 \oplus 0 = 0 \\
C3 &= D2 \oplus D3 \oplus D4 \oplus D8 = 0 \oplus 1 \oplus 1 \oplus 0 = 0 \\
C4 &= D5 \oplus D6 \oplus D7 \oplus D8 = 1 \oplus 1 \oplus 0 \oplus 0 = 0 \\
\end{align*}
\]

• So, check bits are 0001.
Exercise 2

- Suppose that initial check bits are C1=0, C2=1, C3=1 and C4=1.

- When check bits are recalculated for the 8-bit word stored in memory, we have C1=1, C2=1, C3=0 and C4=1.

- Is there any error in the 8-bit word stored in memory? If there is, which bit position has an error?

Exercise 2 - Solution

Initial check bits: C1=0, C2=1, C3=1, C4=1.
Recalculated check bits: C1=1, C2=1, C3=0, C4=1.

<table>
<thead>
<tr>
<th>C4</th>
<th>C3</th>
<th>C2</th>
<th>C1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ \oplus \]

|    | 1  | 0  | 1  | 1  |
|----|----|----|----|
|    | 0  | 1  | 0  | 1  |

- Yes, there is an error in 8-bit word stored in memory.
- Data bit in position 5 (0101) has an error.